

### Stepping Stones for Introduction to Algebra

### Section I - Introduction to Algebra

1) Play 'Think of a number' game -

WORDS	PICTURES	SIMPLIFIED PICTURES	SHORTHAND
Think of a number	8	8	x
Add 3	8000	B+3	x+3
Double	88:00	28+6	2x+6
Take away 4	8800	28+2	2x+2
Divide by 2	8.	8+1	x+1
ake away original nu	imber o	1	1

- Think of a number (single digit number between 1 and 9) Add 3 Multiply by 2 Add your original number Divide by 3 Subtract your original number
- 2) Represent the above problem on slates by using paper cups for the number thought and using unit cubes. Establishing that operations can be done on unknown numbers. We can add, subtract, multiply and divide unknown numbers the way we add, subtract, multiply and divide known numbers.
- 3) Write the problem on blackboard by using pictures of *Gonis* and pictures of blocks.
- 4) Write the problem on blackboard using a combination of picture and numbers.
- 5) Write the problem on board using *x* and the numbers.
- 6) Repeat steps 1 to 5 for the following activity. Think of a number (single digit number between 1 and 9) Add 1 Multiply by 3 Add your original number Add 1 Divide by 4

Subtract your original number

7) Make the following expressions on a slate using paper cups and unit cubes



8) Use a clamp to join two cups by multiplication. So, two cups joined with clamps is  $x^2$ 



Now make the following expressions -

- i) X<sup>2</sup>
- ii) X<sup>2</sup>+ 3
- iii) Y<sup>2</sup>
- iv)  $Y^2 + 5$
- v)  $X^2 + Y^2 + 3$
- vi)  $2x^2 + 3y^2 + 4$
- 9) Make the numbers as per given description on a slate using paper cups and unit cubes -
  - A number greater than x by 3
  - A number which is double of x
  - A number by adding 4 to x
  - Three times x
- 10) Using paper cups and unit make the first bracket on a slate and add to it the second bracket. Write the answer in numerals and symbols.
  - (2X + Y) + (X + Y + 1)
  - (X+1) + (Y+2)

- 11) Using paper cupes and cubes make the first bracket on a slate, subtract the second bracket from the same and write the answer in numerals and symbols -
  - (4 X) (2 X)
  - (3X+5) (X+1)
  - (7Y+3) (5Y)
  - (3X+5Y+4) (X+Y+4)

# 12) Module on negative numbers : (Detailed module attached separately as signed integers)

Representations :

Slate = bank account Green plug = one rupee in the account = +1 (Read as positive one) Red plug = a loan of one rupee in the account = -1 (Read as negative one) A pair of green and red plugs = zero Empty slate = bank account having zero value A slate having same number of green and red plugs = slate having those many zeros = Account having zero value.

We use + and – signs with two meanings. The sign attached to a number tells us whether the number is positive or negative, whether we have a rupee with us or a loan of one rupee with us. The + and – signs between the numbers are used for the process of addition or subtraction. e.g.

(+5) + (-2) should be read as positive five plus negative two. The first number is rupees, second number is loan and we have to add them together.

13) Make the following bank accounts : Find the net value of the account in each case-

- 1) 2 rupees.
- 2) a loan of 2 rupees.
- 3) +5
- 4) -3
- 5) Positive 6
- 6) Negative 4
- 7) +5+3
- 8) -5-3
- 9) +5-3
- 10) -5+3
- 11) +3-3
- 12) -4+4
- 13) 0
- 14) Zero net value, but the slate is not empty

Account A	Account B
0	+3
-2	0
+5	+3
-5	-3
+5	-3
-5	+3
+2+5	-2-5
+2-5	-2+5
0	+3-3

14) Make the following accounts A and B. Compare the net value. (which account will you take if you have to choose between the two accounts) :

15) Add, subtract, multiply and divide the positive and negative numbers. How to do it, how to help children to discover the rules for themselves is given in the attached booklet.

### Section 2 -

### Learn the rules using following examples :

+2+3 = +1+1+1+1+1+1=+5

(Both positive  $\rightarrow$  add the numbers and give positive sign)

+ 3 - 2 = + 1 +  $\cancel{1}$  +  $\cancel{1}$  -  $\cancel{1}$  -  $\cancel{1}$  = +1 (explain cancellation as zero)

(One positive and one negative  $\rightarrow$  Take difference of the two numbers and give sign of the bigger number)

-3 - 2 = -1 - 1 - 1 - 1 - 1 - 1 = -5

(Both negative  $\rightarrow$  add the numbers and give negative sign)

1.	2 + 2	2.	5 + 3	3.	7 + 0	4.	3 – 2
5.	3 + 2	6.	- 3 - 2	7.	-3+3-4 (	cancella	ation as zero)
8.	- 2 - 3 - 5	9.	- 3 - 4 + 7	10.	-24+10+12		

If there is a plus sign outside a bracket  $\rightarrow$  Remove the bracket by keeping signs of the inner terms the same.

1. -3 + (-2) 2. -5 + (+3)

If there is a minus sign outside a bracket  $\rightarrow$  Change the signs of all inner terms.

1. -5-(-3) 2. +5-(+2)

Use the above rules and find the values of the following:

1.	10 + (-7) - (-3)	2.	-4 + (-2) - (-3)
3.	+5 + (+2-4)	4.	-2 - ( - 2 - 3)

### Like terms add.

1.	2x + 2x	2.	5y + 3	y	3.	7a + 0
4.	3x- 2x	5.	$3y^{2} +$	2y <sup>2</sup>	6.	– 3a – 2b
7.	– 3a + 2b – 4	а	8.	- 2x <sup>2</sup> -	- 3x - 5x	x
9.	– 3a – 4a + 7a	a	10.	-24x +	10x + 1	l2x
11	3a – 2a + 5b	– 2b	12.	3a – 21	o + 5a -	- 3b

### **Rules for multiplication**

(+) × (+) = + (product) (-) × (+) = - (product) (+) × (-) = - (product) (-) × (-) = + (product)

### **SOLVE** :(Use the rules to decide the sign and use calculator for multiplication)

1. $(+3) \times (+2)$	2. (+4)×(+5)	3.5×(-2)
4. (-6) × (-9)	5.9×(-12)	6. (-24)x ×10x
7. (-14)×(-1)×7	8. (-50)×2×4	9. (-6)×12×(-2)
10. 9a × (-1)b	11. (-1)a × 9a	12. (-1)×(-1)
13. 15y × 0	14. $(-7)z \times 0$	15. 0.5 × 0.3
$16.\frac{7}{3} \times \frac{(-1)}{2}$	$17.\frac{7a}{3a} \times \frac{(-1)a}{2}$	

### Rules for Division -

+	+	-	_
<b>-</b> = +	- = -	- = -	- = +
+	-	+	-

Solve :

1. $\frac{+6}{+2}$	1. $\frac{+6}{+2}$ 2.		$\frac{3}{3}$ (explain cancellation as 1) 3.		1) 3.	<u>12</u> 6
4.	$\frac{-3}{+3}$		5.	$\frac{-24}{-12}$	6.	$\frac{+6}{-6}$
7.	$\frac{-6}{+2}$		8.	$\frac{-14a}{+7}$	9.	$\frac{-14x^2}{+2x}$

10.	<u>99y</u> 18y	11.	$\frac{-2x^2y}{14xy^2}$	12.	$\frac{7x^3y^2}{-21y^3x^2}$

## Multiplication and addition

(underline the terms first)

1.	2×3+2×3	2.5×5+3×3	3.8×3+12×2
4.	3×3-2×2	53×(-3)+2×(-2)	63×3-2×2
7.	-4×3+1×2×(-3)+12×(-1)	8.6×(-2)+12	95×(-5)+4×4
10	. 4a ×(5a+2)	113x × (6x-2y)	12. (4+5) x (2+1)
13	$.4 + 5 \times 2 + 1$	$14.\frac{3}{4} \times (7-3)$	$154 \times \frac{3}{4} + 2$

### Homework

1.	-48+50-10	2.52-64+1	1	3.72-100+22
4.	12×(-2)	5.(-6)×(-1	2)×(-2)	6.(+6)×(-2)×(-3)
7.	(-1)×8×(-7)	8. (-6)×(-3)	)×(-6)×(-2)×(-	2)
9.	$\frac{-14\times2}{+2\times(-7)}$	10. $\frac{+4\times (2)}{-2\times (2)}$	( <u>-8)</u> (-4)	
11	$\frac{-14x \times 2y}{+2x \times (-7y)}$	12.	$\frac{+4x^2 \times (-8x)}{-2x \times (-4x)}$	

### Section III – Substitution of values in simple expressions :

Make the following expressions using paper cups and red-green plugs.

Use paper cups of two sizes , with colours and plugs of two colours.



Find the values of the following expressions. Use calculator if necessary.

1)	Find	x + 3,	if $x = 2$
2)	Find	7 – x	if $x = 3$
3)	Find	x + 5	if x = 0
4)	Find	x – 4	if $x = 10$
5)	Find	x <sup>2</sup>	if $x = 25$
6)	Find	x <sup>3</sup>	if $x = 8$

If x = 3 and y = 5, find the values of following expressions :

x + y
2x + y
2x – y
3x + 2y
3x - 2y
3x - 4y

Find the value of 3x - 4y if x = 4 and y = 3

### Section IV - Simple Equations

Write the following pictures in algebraic terms and solve : (A cup holds unknown number x, y, a, b or whatever you call it.)

To solve means to find the value of x, to find what is there inside the cup. For that you will have to keep only cup on the left.



(Subtract 1 from both sides, then divide both sides by 2)

Solve the following equations by using the rules.

1) 3)	$ \begin{array}{l} \mathbf{x} + 2 = 4 \\ 2\mathbf{x} = 8 \end{array} $	2) 4)	$\begin{array}{l} x - 2 = 3\\ 2x + 1 = 7 \end{array}$		
5)	$\frac{x}{3} = 4$	6)	$\frac{2x}{3} = 4$		
7)	$\frac{x}{2} + 1 = 5$	8)	$\frac{2x}{3} + 4 = 7$		
9)	$\frac{3x+4}{2} = 5$	10)	$\frac{3x}{2} + 4 = 10$		
11)	$x + \frac{1}{2} = 1$	12)	$2x + \frac{3}{4} = \frac{3}{2}$		
Homework :					
Find th	ne value of x				

1.	x+2=4	2.	x - 2=3	3.	x - 5=0
4.	x+12=24	5.	2x=6	6.	2x+4=10
7.	3x+9=27	8.	3x-5x+4=2	9.	6x-2x+5=8

10. 
$$\frac{x}{3} = 4$$
 11.  $\frac{2x}{3} = 4$  12.  $\frac{3x}{9} = 3$ 

13. 
$$\frac{x}{2} + 1 = 5$$
 14.  $\frac{2x}{3} + 2 = 4$  15.  $\frac{x}{2} + 3 = 5$ 

## Section IV - Equations and Substitution :

1. 
$$x+y=0$$
, if  $x=0$ , find y2.  $2x+3y=0$ , if  $x=0$ , find y3.  $3x+2y=12$ , if  $x=0$ , find y3.  $3x+2y=12$ , if  $x=0$ , find y4.  $2x+6y=18$ , if  $x=0$ , find y5.  $x+2y=6$ , if  $x=0$ , find y5.  $x+2y=6$ , if  $x=0$ , find y6.  $2x+5y=10$ , if  $x=0$ , find y7.  $x+y=3$ , if  $x=0$ , find y9.  $x+y=3$ , if  $x=0$ , find y

Use the value of  $\pi = \frac{22}{7}$ 

8. $c = 2\pi r$ , $\ln r = 7$ , $\ln n c$	8.	$c = 2\pi r$	,if r=7, find ₀
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9. $A = \pi r^2$	,if r=7, find A
$10.A = 2\pi rh$	, if r=14, h=4,find A
$11.V = \frac{4}{3}\pi r^3$	, if r=21, find V
12. $A = 4\pi r^2$	, if r=14, find A
13. $A = \pi r l$	, if r=14, l=18, find A
$14. A = 3\pi r^2$	, if r=7, find A

### HOMEWORK

1. 5x-2x+5=82. 6x-2x-24=-123. -5x+4=94.  $\frac{2x}{3}+4=7$ 5.  $\frac{3x}{2}+2=8$ 6. x+2y=6, if x=0, find y if y=0, find x 7. 2x+5y=10, if x=0, find y if y=0, find x 9. x+y=3, if x=0, find y if y=0, find x Use the value of  $\pi = \frac{22}{7}$ 10.  $A = \pi r l + \pi r^2$ , if r=14, l=18, find A 11.  $A = 2\pi r h + 2\pi r^2$ , if r=14, h=4, find A 12.  $A = 2\pi r^2 + \pi r^2$ , if r=14, find A 13. 2x + 7 = 114. 3x - 1 = 115. x + y = 7 Find the all possible values of pairs , if  $x, y \ge 0$ 

### Word Problems:

- 1. Devika had some books. Her father presented her 5 books. She lost 2 books . Now she has 15 books. How many books had she initially?
- 2. I had 3 boxes, each containing same number of pens. After giving 5 pens to my friends, I have 7 pens. Find the number of pens in each box.

# Section V - Multiplication by rectangles and Identities : (Use the rules of multiplying positive and negative numbers)

### On Grid notebook :

- 1. 4 × 3
- $2. 10 \times 10$
- 3. 12 × 13

### On blank paper :

- 4.  $15 \times 18$
- 5. a × a
- 6.  $3 \times x$
- 7. a × b

### Identities

- 1) Take a square piece from Identity kit. If the side is a, what is the area?
- 2) Take another square piece from Identity kit. If the side is b, what is the area?
- 3) Take a rectangular piece from Identity kit. If one side is a, the other side is b, what is the area?
- 4) Take a<sup>2</sup>, b<sup>2</sup>, and 2 pieces of ab from identity kit. Join them to make a bigger square.
- 5) Do you see that  $(a+b)^2 = a^2+2ab+b^2$ ?
- 6) Verify Pythagoras theorem using identity kit.

### Draw rectangles

- 8.  $(a+b) \times c$
- 9.  $a \times (b + c)$
- 10.  $(a+b)^2 = (a+b) \times (a+b)$
- 11.  $(a b)^2 = (a b) \times (a b)$
- 12.  $(a+b) \times (a-b)$
- 13. (x + a) (x + b)

### LEARN BY HEART

1) a(x + y) = ax + ay2) a(x - y) = ax - ay3)  $(x + y)^2 = x^2 + 2xy + y^2$ 4)  $(x - y)^2 = x^2 - 2xy + y^2$ 5)  $(x + y)(x - y) = x^2 - y^2$ 6)  $(x + a)(x + b) = x^2 + (a+b)x + b^2$  In all of the above, the left hand side is a product (or multiplication), also called as factorisation. Right hand side is called expansion (it is expression)

SOLVE :

1)	$(x + y)^2$
2)	$(2x + 3y)^2$
3)	(a – b) <sup>2</sup>
4)	(3x - y) <sup>2</sup>
5)	$(x+2)^2$
6)	(a – 3) <sup>2</sup>
7)	(a+b)(a-b)
8)	(x+y)(x-y)
9)	(2a+3b)(2a-3b)
10)	$(a + \sqrt{2})^2$
11)	$(a - \sqrt{3})^2$
12)	$(\sqrt{2} + \sqrt{3})^2$

Solve :

1.	(x+5)(x+12)	2.	(x+8)(x+2)
3.	(y+3)(y+15)	4.	(n + 12)(n + 15)
5.	(x + 0.4) (x + 0.5)	6.	(y + 0.7)(y + 0.1)
7.	$\left(x+\frac{1}{3}\right)\left(x+\frac{2}{3}\right)$	8.	$\left(y + \frac{4}{5}\right)\left(y + \frac{2}{5}\right)$
9.	(x+4)(x-1)	10.	(x + 15)(x - 8)
11.	(m+8)(m-8)	12.	(n+15)(n-12)
13.	(x + 0.8)(x - 0.5)	14.	(y + 0.7)(y - 0.1)
15.	$\left(x+\frac{3}{4}\right)\left(x-\frac{1}{4}\right)$	16.	$\left(y+\frac{4}{5}\right)\left(y-\frac{2}{5}\right)$

17. 
$$(x-8)(x+5)$$
  
18.  $(x-12)(x+7)$   
19.  $(m-5)(m+5)$   
20.  $(x-18)(x+3)$   
21.  $(x-1.6) \times (x+0.9)$   
22.  $\left(x - \frac{7}{15}\right) \left(x + \frac{2}{15}\right)$   
23.  $\left(y - \frac{4}{5}\right) \left(y + \frac{3}{5}\right)$   
24.  $(x-7)(x-4)$   
25.  $(x-5)(x-20)$   
26.  $(m-9)(m-9)$   
27.  $(x-0.7)(x-1.4)$   
28.  $(y-2.4)(y-0.6)$   
29.  $\left(x - \frac{4}{9}\right) \left(x - \frac{5}{9}\right)$   
30.  $\left(y - \frac{7}{13}\right) \left(y - \frac{3}{13}\right)$   
HOMEWORK

i) 
$$(x + 15)(x + 9)$$
  
ii)  $(x + 15)(x - 9)$   
iii)  $(x - 15)(x + 9)$   
iv)  $(x - 15)(x - 9)$   
v)  $(y + 0.8)(y + 0.4)$   
vi)  $(y - 0.8)(y - 0.4)$   
viii)  $(y - 0.8)(y - 0.4)$   
viii)  $(z + \frac{2}{5})(z - \frac{1}{5})$   
x)  $(z - \frac{2}{5})(z - \frac{1}{5})$   
iv)  $(z - \frac{2}{5})(z - \frac{1}{5})$ 

### Section VI - Factorisation

#### (draw rectangles to show factors) :



5) 2a + 2b (learn to take out common term)

6)  $x^2 + (a+b)x + ab$ 

If the sign of term ab is positive, a and b both are either positive or both are negative. In that case the coefficient of middle term is the addition of those two numbers. If middle term is positive, a and b both are positive. If middle term is negative, both a and b are negatve.

If the sign of term ab is negative, one of them is positive and the other one is negative. In that case the coefficient of middle term is the difference of two numbers. Sign of the middle term is sign of bigger term.

e.g.  $x^2+3x+2$ 

find two numbers of which product is 2 and sum is 3

Section VII -	Factorisation	: of the type	$: x^2 - y^2 =$	(x+y)(x-y)

Find the value of squares using calculator :

$1^2 =$	$11^2 =$
$2^2 =$	$12^2 =$
$3^2 =$	$13^2 =$
$4^2 =$	$14^2 =$
$5^2 =$	$15^2 =$
$6^2 =$	$16^2 =$
$7^2 =$	$17^2 =$
$8^2 =$	$18^2 =$
$9^2 =$	$19^2 =$

 $10^2 = 20^2 =$ 

Find the squares of :

1.	2a	2. $\frac{a}{2}$	3.	$\frac{1}{10}$
4.	$\frac{1}{4}$	5. $\frac{1}{x}$	6.	$\frac{1}{x^2}$

Find the square-roots of :

1.	<i>x</i> <sup>2</sup>	2.	$4x^2$	3.	$9x^4$
4.	<u>4</u> 9	5.	$\frac{1}{x^2}$	6.	$\frac{x^9}{4}$

### Fill in the blanks :

1.	$49x^2 = ()^2$	2.	$36x^2 = ()^2$	3.	$25a^2 = ()^2$
4.	$144a^2 = ()^2$	5.	$81y^2 = ()^2$	6.	$\frac{1}{x^2} = ()^2$
7.	$\frac{9}{x^2} = ()^2$	8.	$\frac{a^2}{16} = ()^2$	9.	$\frac{a^2}{b^2} = ()^2$

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10.	$\frac{16}{9x^2} = ()^2$		11.	$\frac{225b^2}{a^2} = ()^2$	2	12.	$\frac{196x^2}{49} = ()^2$
13.	$\frac{16a^2}{25b^2} = ()^2$		14.	$\frac{4}{x^2} = ()^2$		15.	$\frac{144a^2}{169} = ()^2$
Facto	rise :						
1.	x <sup>2</sup> -y <sup>2</sup>	2.	a <sup>2</sup> -b <sup>2</sup>		3.	a <sup>2</sup> -9b <sup>2</sup>	2
4.	4a <sup>2</sup> -b <sup>2</sup>	5.	4p²-9	q <sup>2</sup>	6.	49x <sup>2</sup> -3	36y <sup>2</sup>
7.	$x^2 - \frac{1}{x^2}$	8.	25m <sup>2</sup>	-9n <sup>2</sup>	9.	81p <sup>2</sup> -	q <sup>2</sup>
10.	$225x^2-144y^2$	11.	100a <sup>2</sup>	<sup>2</sup> – 1	12.	64p² -	121

# SQUARE ROOT

 $x^{2}=9$ x=± $\sqrt{9}$ = +3 or -3 = ±3

Find the value x

1. $x^2 = 64$	1	2.	x <sup>2</sup> =81	3.	$2x^2=32$
4. $4x^2 = 4$	1	5.	$2x^2-5=13$	6.	$3x^2-6=42$
7. $5x^2 + 2$	25 = 200	8.	$2x^2 + 2 = 100$	9.	$x^2 - 64 = 0$
10.	$4x^2 - 6 = 30$				

### Section VIII - Exam questions :

- If x<sup>2</sup> + ax + b = (x + 2) (x 2), find a and b.
   (expand the right hand side and by observation find a and b)
- If (x-3)(x+a) = x<sup>2</sup>-px+12, find a and p.
  (expand the left hand side and by observation find a and p)
- 3. If one root of the equation  $2x^2-7x+k=0$  is 3, find k. (substitute x as 3 and find the value of k)
- 4. If one root of the equation  $2x^2 7x + k = 0$  is  $\frac{1}{2}$  find the value of k. (substitute x as 1/2 and find the value of k)
- 5. For what values of k, the equations kx + 2y = 0 and (k-1)x + 3y = 12 will have no solution? (Ratio of coefficients of x and ratio of coefficients of y is equal)
- 6. For what value of k, the equations 2x + ky = 11 and 5x 7y = 5 will have no solution? (Ratio of coefficients of x and ratio of coefficients of y is equal)
- 7. If x = 2t and y = t/3 1, for what values of t, x = 2y?
  (Substitute the values of x and y given in terms of t, into the equation x = 2y and find t)
- 8. If the ordered pair (3y-x, 3-x) and (x+y, 2y) are identical, find x and y.
- 9. If the sides of a right angle triangle are x+1, x+2 and x+3 units find x.

## Section IV - Equations and Substitution :

15. x + y = 0	, if x=0, find y
	if y=0, find x
16.2x+3y=0	, if x=0, find y
	if y=0, find x
17.3x+2y=12	, if x=0, find y
	if y=0, find x
18.2x+6y=18	,if x=0, find y
	if y=0, find x
19.x + 2y = 6	, if x=0, find y
	if y=0, find x
20.2x+5y=10	,if x=0, find y
	if y=0, find x
21. x+y=3	,if x=0, find y
	if y=0, find x

Use the value of  $\pi = \frac{22}{7}$ 

$22. c = 2\pi r$	,if r=7, find c
$23.A = \pi r^2$	,if r=7, find A
$24. A = 2\pi rh$	, if r=14, h=4,find A
$25.V = \frac{4}{3}\pi r^3$	, if r=21, find V
$26.A = 4\pi r^2$	, if r=14, find A
$27.A = \pi rl$	, if r=14, l=18, find A
28. $A = 3\pi r^2$	, if r=7, find A

### HOMEWORK

1.5x-2x+5=8	2.6x-2x-24 = -12	35x+4=9
$4.\frac{2x}{3}+4=7$	$5.\frac{3x}{2}+2=8$	
6. x+2y=6	, if x=0, find y	
	if y=0, find x	
7. 2x+5y=10	, if x=0, find y	
	if y=0, find x	
9. x+y=3	, if x=0, find y	
	if y=0, find x	

Use the value of  $\pi = \frac{22}{7}$ 

- 10.  $A = \pi r l + \pi r^2$ , if r=14, l=18, find A 11.  $A = 2\pi r h + 2\pi r^2$ , if r=14, h=4,find A 12.  $A = 2\pi r^2 + \pi r^2$ , if r=14, find A 13. 2x + 7 = 114. 3x - 1 = 1
- 15. x + y =7 Find the all possible values of pairs , if x,y  $\geq$  0

### Word Problems:

- 3. Devika had some books. Her father presented her 5 books. She lost 2 books . Now she has 15 books. How many books had she initially?
- 4. I had 3 boxes, each containing same number of pens. After giving 5 pens to my friends, I have 7 pens. Find the number of pens in each box.